# Mark Scheme (Results)
# January 2008

# GCE Mathematics (6666/01)

| Question Number | Scheme | Marks |
|---|---|---|
| **1.** (a) | | |

| $x$ | 0 | $\frac{\pi}{4}$ | $\frac{\pi}{2}$ | $\frac{3\pi}{4}$ | $\pi$ |
|---|---|---|---|---|---|
| $y$ | 0 | 1.844321332… | 4.810477381… | 8.87207 | 0 |

awrt 1.84432   B1
awrt 4.81048 or 4.81047   B1

**[2]**

Outside brackets
awrt 0.39 or $\frac{1}{2} \times$ awrt 0.79   B1
$\frac{1}{2} \times \frac{\pi}{4}$ or $\frac{\pi}{8}$

0 can be implied

(b)
**Way 1**

Area $\approx \dfrac{1}{2} \times \dfrac{\pi}{4}$ ; $\times \left\{ 0 + 2(1.84432 + 4.81048 + 8.87207) + 0 \right\}$

For structure of trapezium rule $\underline{\{............\}}$ ;   M1 $\sqrt{}$

Correct expression inside brackets which all must be multiplied by their "outside constant".   A1 $\sqrt{}$

$= \dfrac{\pi}{8} \times 31.05374... = 12.19477518... = \underline{12.1948}$ (4dp)    12.1948   A1 **cao**

**[4]**

*Aliter*
(b)
**Way 2**

Area $\approx \dfrac{\pi}{4} \times \left\{ \dfrac{0+1.84432}{2} + \dfrac{1.84432+4.81048}{2} + \dfrac{4.81048+8.87207}{2} + \dfrac{8.87207+0}{2} \right\}$

$\frac{\pi}{4}$ (or awrt 0.79 ) and a divisor of 2 on all terms inside brackets.   B1

which is equivalent to:

One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2.   M1 $\sqrt{}$

Area $\approx \dfrac{1}{2} \times \dfrac{\pi}{4}$ ; $\times \left\{ 0 + 2(1.84432 + 4.81048 + 8.87207) + 0 \right\}$

Correct expression inside brackets if $\frac{1}{2}$ was to be factorised out.   A1 $\sqrt{}$

$= \dfrac{\pi}{4} \times 15.52687... = 12.19477518... = \underline{12.1948}$ (4dp)    12.1948   A1 **cao**

**[4]**

**6 marks**

Note an expression like Area $\approx \dfrac{1}{2} \times \dfrac{\pi}{4} + 2(1.84432 + 4.81048 + 8.87207)$ would score B1M1A0A0

| Question Number | Scheme | Marks |
|---|---|---|
| **2.** (a) | ** represents a constant<br>(which must be consistent for first accuracy mark)<br><br>$(8-3x)^{\frac{1}{3}} = \underline{(8)^{\frac{1}{3}}}\left(1-\frac{3x}{8}\right)^{\frac{1}{3}} = \underline{2}\left(1-\frac{3x}{8}\right)^{\frac{1}{3}}$    Takes 8 outside the bracket to give any of $\underline{(8)^{\frac{1}{3}}}$ or $\underline{2}$ . | <u>B1</u> |
| | $= 2\left\{1+(\tfrac{1}{3})(**\,x); + \frac{(\tfrac{1}{3})(-\tfrac{2}{3})}{2!}(**\,x)^2 + \frac{(\tfrac{1}{3})(-\tfrac{2}{3})(-\tfrac{5}{3})}{3!}(**\,x)^3 + ...\right\}$ <br><br> **with $** \neq 1$**    Expands $(1+**\,x)^{\frac{1}{3}}$ to give a simplified or an un-simplified $1+(\tfrac{1}{3})(**\,x)$ ; <br> A correct simplified or an un-simplified $\{.........\}$ expansion with candidate's followed through $(**\,x)$ | M1;<br><br>A1$\sqrt{}$ |
| | $= 2\left\{1+(\tfrac{1}{3})(-\tfrac{3x}{8}) + \frac{(\tfrac{1}{3})(-\tfrac{2}{3})}{2!}(-\tfrac{3x}{8})^2 + \frac{(\tfrac{1}{3})(-\tfrac{2}{3})(-\tfrac{5}{3})}{3!}(-\tfrac{3x}{8})^3 + ...\right\}$    Award SC M1 if you see $\frac{(\tfrac{1}{3})(-\tfrac{2}{3})}{2!}(**\,x)^2 + \frac{(\tfrac{1}{3})(-\tfrac{2}{3})(-\tfrac{5}{3})}{3!}(**\,x)^3$ | |
| | $= 2\left\{1-\tfrac{1}{8}x; -\tfrac{1}{64}x^2 - \tfrac{5}{1536}x^3 - ...\right\}$ <br><br> $= 2 - \frac{1}{4}x; \; -\frac{1}{32}x^2 - \frac{5}{768}x^3 - ...$    Either $2\left\{1-\tfrac{1}{8}x ........\right\}$ or anything that cancels to $2-\tfrac{1}{4}x;$ <br> Simplified $-\tfrac{1}{32}x^2 - \tfrac{5}{768}x^3$ | A1;<br><br>A1<br>**[5]** |
| (b) | $(7.7)^{\frac{1}{3}} \approx 2 - \frac{1}{4}(0.1) - \frac{1}{32}(0.1)^2 - \frac{5}{768}(0.1)^3 - ...$    ***Attempt to substitute*** $x=0.1$ into a candidate's binomial expansion. | M1 |
| | $= 2 - 0.025 - 0.0003125 - 0.0000065104166...$ <br><br> $= 1.97468099...$    awrt 1.9746810 | A1<br>**[2]** |
| | | **7 marks** |

You would award B1M1A0 for

$$= 2\left\{1+(\tfrac{1}{3})(-\tfrac{3x}{8}) + \frac{(\tfrac{1}{3})(-\tfrac{2}{3})}{2!}(-\tfrac{3x}{8})^2 + \frac{(\tfrac{1}{3})(-\tfrac{2}{3})(-\tfrac{5}{3})}{3!}(-3x)^3 + ...\right\}$$

because ** is not consistent.

If you see the constant term "2" in a candidate's final binomial expansion, then you can award B1.

Be wary of calculator value of $(7.7)^{\frac{1}{3}} = 1.974680822...$

3

| Question Number | Scheme | Marks |
|---|---|---|
| *Aliter* <br> **2.** (a) <br> **Way 2** | $(8-3x)^{\frac{1}{3}}$ <br><br> $2$ or $(8)^{\frac{1}{3}}$ (**See note** $\downarrow$) <br><br> $= \left\{ \begin{array}{l} (8)^{\frac{1}{3}} + (\frac{1}{3})(8)^{-\frac{2}{3}}(**x); + \dfrac{(\frac{1}{3})(-\frac{2}{3})}{2!}(8)^{-\frac{5}{3}}(**x)^2 \\ \qquad + \dfrac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(8)^{-\frac{8}{3}}(**x)^3 + ... \end{array} \right\}$ <br><br> **with** $** \neq 1$ <br><br> $= \left\{ \begin{array}{l} (8)^{\frac{1}{3}} + (\frac{1}{3})(8)^{-\frac{2}{3}}(-3x); + \dfrac{(\frac{1}{3})(-\frac{2}{3})}{2!}(8)^{-\frac{5}{3}}(-3x)^2 \\ \qquad + \dfrac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(8)^{-\frac{8}{3}}(-3x)^3 + ... \end{array} \right\}$ <br><br> $= \left\{ 2 + (\frac{1}{3})(\frac{1}{4})(-3x) + (-\frac{1}{9})(\frac{1}{32})(9x^2) + (\frac{5}{81})(\frac{1}{256})(-27x^3) + ... \right\}$ <br><br> $= 2 - \dfrac{1}{4}x; - \dfrac{1}{32}x^2 - \dfrac{5}{768}x^3 - ...$ | 2 or $(8)^{\frac{1}{3}}$ (**See note** $\downarrow$)    B1 <br><br> Expands $(8-3x)^{\frac{1}{3}}$ to give an un-simplified or simplified    M1; <br> $(8)^{\frac{1}{3}} + (\frac{1}{3})(8)^{-\frac{2}{3}}(**x);$ <br> A correct un-simplified or simplified $\{.........\}$ expansion with candidate's followed through $(**x)$    A1 $\sqrt{\phantom{x}}$ <br><br> Award SC M1 if you see <br> $\dfrac{(\frac{1}{3})(-\frac{2}{3})}{2!}(8)^{-\frac{5}{3}}(**x)^2$ <br> $+ \dfrac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(8)^{-\frac{8}{3}}(**x)^3$ <br><br> Anything that cancels to $2 - \dfrac{1}{4}x;$    A1; <br> or $2\{1 - \frac{1}{8}x ........\}$ <br> Simplified $-\frac{1}{32}x^2 - \frac{5}{768}x^3$    A1 <br> **[5]** |

Attempts using Maclaurin expansion should be escalated up to your team leader.

Be wary of calculator value of $(7.7)^{\frac{1}{3}} = 1.974680822...$

If you see the constant term "2" in a candidate's final binomial expansion, then you can award B1.

| Question Number | Scheme | Marks |
|---|---|---|
| **3.** | Volume $= \pi \int_a^b \left( \dfrac{1}{2x+1} \right)^2 \mathrm{d}x = \pi \int_a^b \dfrac{1}{(2x+1)^2} \mathrm{d}x$      Use of $V = \pi \int y^2 \, \mathrm{d}x$. Can be implied. Ignore limits. | B1 |
| | $= \pi \int_a^b (2x+1)^{-2} \, \mathrm{d}x$ | |
| | $= (\pi) \left[ \dfrac{(2x+1)^{-1}}{(-1)(2)} \right]_a^b$ | |
| | $= (\pi) \left[ -\tfrac{1}{2}(2x+1)^{-1} \right]_a^b$      Integrating to give $\pm p(2x+1)^{-1}$    $-\tfrac{1}{2}(2x+1)^{-1}$ | M1<br>A1 |
| | $= (\pi) \left[ \left( \dfrac{-1}{2(2b+1)} \right) - \left( \dfrac{-1}{2(2a+1)} \right) \right]$    Substitutes limits of $b$ and $a$ and subtracts the correct way round. | dM1 |
| | $= \dfrac{\pi}{2} \left[ \dfrac{-2a-1+2b+1}{(2a+1)(2b+1)} \right]$ | |
| | $= \dfrac{\pi}{2} \left[ \dfrac{2(b-a)}{(2a+1)(2b+1)} \right]$ | |
| | $= \dfrac{\pi(b-a)}{(2a+1)(2b+1)}$      $\dfrac{\pi(b-a)}{(2a+1)(2b+1)}$ | A1 aef<br>**[5]** |
| | | **5 marks** |

Allow other equivalent forms such as

$$\dfrac{\pi b - \pi a}{(2a+1)(2b+1)} \quad \text{or} \quad \dfrac{-\pi(a-b)}{(2a+1)(2b+1)} \quad \text{or} \quad \dfrac{\pi(b-a)}{4ab+2a+2b+1} \quad \text{or} \quad \dfrac{\pi b - \pi a}{4ab+2a+2b+1}.$$

Note that $\pi$ is not required for the middle three marks of this question.

| Question Number | Scheme | Marks |
|---|---|---|
| **Aliter 3. Way 2** | Volume $= \pi \int_a^b \left( \frac{1}{2x+1} \right)^2 \mathrm{d}x = \pi \int_a^b \frac{1}{(2x+1)^2} \mathrm{d}x$    Use of $V = \pi \int y^2 \, \mathrm{d}x$. Can be implied. Ignore limits. | B1 |
| | $= \pi \int_a^b (2x+1)^{-2} \, \mathrm{d}x$ | |
| | Applying substitution $u = 2x+1 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2$ and changing limits $x \to u$ so that $a \to 2a+1$ and $b \to 2b+1$ , gives | |
| | $= (\pi) \int_{2a+1}^{2b+1} \frac{u^{-2}}{2} \, \mathrm{d}u$ | |
| | $= (\pi) \left[ \frac{u^{-1}}{(-1)(2)} \right]_{2a+1}^{2b+1}$ | |
| | $= (\pi) \left[ -\frac{1}{2} u^{-1} \right]_{2a+1}^{2b+1}$    Integrating to give $\pm p u^{-1}$   $-\frac{1}{2} u^{-1}$ | M1 A1 |
| | $= (\pi) \left[ \left( \frac{-1}{2(2b+1)} \right) - \left( \frac{-1}{2(2a+1)} \right) \right]$   Substitutes limits of $2b+1$ and $2a+1$ and subtracts the correct way round. | dM1 |
| | $= \frac{\pi}{2} \left[ \frac{-2a-1+2b+1}{(2a+1)(2b+1)} \right]$ | |
| | $= \frac{\pi}{2} \left[ \frac{2(b-a)}{(2a+1)(2b+1)} \right]$ | |
| | $= \frac{\pi(b-a)}{(2a+1)(2b+1)}$    $\frac{\pi(b-a)}{(2a+1)(2b+1)}$ | A1 aef **[5]** |
| | | **5 marks** |

Note that $\pi$ is not required for the middle three marks of this question.

Allow other equivalent forms such as

$$\frac{\pi b - \pi a}{(2a+1)(2b+1)} \quad \text{or} \quad \frac{-\pi(a-b)}{(2a+1)(2b+1)} \quad \text{or} \quad \frac{\pi(b-a)}{4ab+2a+2b+1} \quad \text{or} \quad \frac{\pi b - \pi a}{4ab+2a+2b+1}.$$

| Question Number | Scheme | Marks |
|---|---|---|
| **4.** (i) | $\int \ln\left(\frac{x}{2}\right) dx = \int 1.\ln\left(\frac{x}{2}\right) dx \Rightarrow$ $\begin{cases} u = \ln\left(\frac{x}{2}\right) & \Rightarrow \quad \frac{du}{dx} = \frac{\frac{1}{2}}{\frac{x}{2}} = \frac{1}{x} \\ \frac{dv}{dx} = 1 & \Rightarrow \quad v = x \end{cases}$ | |

$$\int \ln\left(\frac{x}{2}\right) dx = x\ln\left(\frac{x}{2}\right) - \int x.\frac{1}{x} dx$$

Use of 'integration by parts' formula in the correct direction. **M1**
Correct expression. **A1**

$$= x\ln\left(\frac{x}{2}\right) - \int 1 \, dx$$

An attempt to multiply $x$ by a candidate's $\frac{a}{x}$ or $\frac{1}{bx}$ or $\frac{1}{x}$. **dM1**

$$= x\ln\left(\frac{x}{2}\right) - x + c$$

Correct integration with $+ c$ **A1 aef**

**[4]**

(ii)  $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx$

$$\left[ \text{NB}: \ \underline{\cos 2x = \pm 1 \pm 2\sin^2 x} \ \text{ or } \ \underline{\sin^2 x = \tfrac{1}{2}\left(\pm 1 \pm \cos 2x\right)} \right]$$

Consideration of double angle formula for $\cos 2x$ **M1**

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1-\cos 2x}{2} \, dx = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(1-\cos 2x\right) dx$$

$$= \frac{1}{2}\Big[ \ \underline{x - \tfrac{1}{2}\sin 2x} \ \Big]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

<u>Integrating to give</u> $\pm ax \pm b\sin 2x \, ; a, b \neq 0$ **dM1**
Correct result of anything equivalent to $\frac{1}{2}x - \frac{1}{4}\sin 2x$ **A1**

$$= \frac{1}{2}\left[ \left(\frac{\pi}{2} - \frac{\sin(\pi)}{2}\right) - \left(\frac{\pi}{4} - \frac{\sin\left(\frac{\pi}{2}\right)}{2}\right) \right]$$

$$= \frac{1}{2}\left[ \left(\frac{\pi}{2} - 0\right) - \left(\frac{\pi}{4} - \frac{1}{2}\right) \right]$$

Substitutes limits of $\frac{\pi}{2}$ and $\frac{\pi}{4}$ and subtracts the correct way round. **ddM1**

$$= \frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right) = \frac{\pi}{8} + \frac{1}{4}$$

$\frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right)$ or $\frac{\pi}{8} + \frac{1}{4}$ or $\frac{\pi}{8} + \frac{2}{8}$ **A1 aef , cso**

**[5]**

Candidate must collect their $\pi$ term and constant term together for A1
No fluked answers, hence **cso**.

**9 marks**

Note: $\int \ln\left(\frac{x}{2}\right) dx = (\text{their } v)\ln\left(\frac{x}{2}\right) - \int (\text{their } v).\left(\text{their } \frac{du}{dx}\right) dx$ for M1 in part (i).

Note $\frac{\pi}{8} + \frac{1}{4} = 0.64269...$

| Question Number | Scheme | Marks |
|---|---|---|
| **Aliter** 4. (i) **Way 2** | $\int \ln\left(\frac{x}{2}\right) dx = \int (\ln x - \ln 2) \, dx = \int \ln x \, dx - \int \ln 2 \, dx$ $\int \ln x \, dx = \int 1 . \ln x \, dx \Rightarrow \begin{cases} u = \ln x & \Rightarrow & \dfrac{du}{dx} = \frac{1}{x} \\ \dfrac{dv}{dx} = 1 & \Rightarrow & v = x \end{cases}$ $\int \ln x \, dx = x \ln x - \int x . \frac{1}{x} \, dx$        Use of 'integration by parts' formula in the correct direction. | M1 |
| | $= x \ln x - x + c$             Correct integration of $\ln x$ with or without $+ c$ | A1 |
| | $\int \ln 2 \, dx = x \ln 2 + c$         Correct integration of $\ln 2$ with or without $+ c$ | M1 |
| | Hence, $\int \ln\left(\frac{x}{2}\right) dx = x \ln x - x - x \ln 2 + c$     Correct integration with $+ c$ | A1 aef |
| | | **[4]** |

Note: $\int \ln x \, dx = (\text{their } v) \ln x - \int (\text{their } v) . (\text{their } \frac{du}{dx}) \, dx$ for M1 in part (i).

| Question Number | Scheme | Marks |
|---|---|---|
| **_Aliter_** **4.** (i) **Way 3** | $\displaystyle\int \ln\left(\tfrac{x}{2}\right)\,\mathrm{d}x$ $u = \tfrac{x}{2} \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x} = \dfrac{1}{2}$ $\displaystyle\int \ln\left(\tfrac{x}{2}\right)\,\mathrm{d}x = 2\int \ln u\,\mathrm{d}u$ Applying substitution correctly to give $\displaystyle\int \ln\left(\tfrac{x}{2}\right)\,\mathrm{d}x = 2\int \ln u\,\mathrm{d}u$ **_Decide to award 2$^{nd}$ M1 here!_** $\displaystyle\int \ln u\,\mathrm{d}x = \int 1.\ln u\,\mathrm{d}u$ $\displaystyle\int \ln u\,\mathrm{d}x = u\ln u - \int u.\tfrac{1}{u}\,\mathrm{d}u$        Use of 'integration by parts' formula in the correct direction. $= u\ln u - u + c$        Correct integration of $\ln u$ with or without $+c$        **_Decide to award 2$^{nd}$ M1 here!_** $\displaystyle\int \ln\left(\tfrac{x}{2}\right)\mathrm{d}x = 2\left(u\ln u - u\right) + c$ Hence, $\displaystyle\int \ln\left(\tfrac{x}{2}\right)\,\mathrm{d}x = x\ln\left(\tfrac{x}{2}\right) - x + c$        Correct integration with $+c$ | M1 A1 M1 A1 aef **[4]** |

| Question Number | Scheme | Marks |
|---|---|---|
| *Aliter* **4.** (ii) **Way 2** | $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x . \sin x \, dx$ and $I = \displaystyle\int \sin^2 x \, dx$ | |

$\begin{cases} u = \sin x & \Rightarrow \frac{du}{dx} = \cos x \\ \frac{dv}{dx} = \sin x & \Rightarrow v = -\cos x \end{cases}$

$\therefore I = \left\{ \underline{-\sin x \cos x + \int \cos^2 x \, dx} \right\}$      An attempt to use the correct by parts formula.    **M1**

$\therefore I = \left\{ -\sin x \cos x + \int \left(1 - \sin^2 x\right) dx \right\}$

$\int \sin^2 x \, dx = \left\{ -\sin x \cos x + \int 1 \, dx - \int \sin^2 x \, dx \right\}$

$2\int \sin^2 x \, dx = \left\{ -\sin x \cos x + \int 1 \, dx \right\}$      For the LHS becoming $2I$    **dM1**

$2\int \sin^2 x \, dx = \left\{ -\sin x \cos x + x \right\}$

$\int \sin^2 x \, dx = \left\{ \underline{-\frac{1}{2}\sin x \cos x + \frac{x}{2}} \right\}$      Correct integration    **A1**

$\therefore \displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx = \left[ \left(-\frac{1}{2}\sin(\frac{\pi}{2})\cos(\frac{\pi}{2}) + \frac{(\frac{\pi}{2})}{2}\right) - \left(-\frac{1}{2}\sin(\frac{\pi}{4})\cos(\frac{\pi}{4}) + \frac{(\frac{\pi}{4})}{2}\right) \right]$

Substitutes limits of $\frac{\pi}{2}$ and $\frac{\pi}{4}$ and subtracts the correct way round.    **ddM1**

$= \left[ (0 + \frac{\pi}{4}) - (-\frac{1}{4} + \frac{\pi}{8}) \right]$

$= \frac{\pi}{8} + \frac{1}{4}$      $\underline{\frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right)}$ or $\underline{\frac{\pi}{8} + \frac{1}{4}}$ or $\underline{\frac{\pi}{8} + \frac{2}{8}}$

**A1 aef cso**

**[5]**

Candidate must collect their $\pi$ term and constant term together for A1
No fluked answers, hence **cso**.

Note $\frac{\pi}{8} + \frac{1}{4} = 0.64269...$

| Question Number | Scheme | Marks |
|---|---|---|
| **5.** (a) | $x^3 - 4y^2 = 12xy$  ( eqn $*$ ) | |

$x = -8 \implies -512 - 4y^2 = 12(-8)y$

$\qquad\qquad -512 - 4y^2 = -96y$

Substitutes $x = -8$ (at least once) into $*$ to obtain a three term quadratic in $y$. Condone the loss of $= 0$. **M1**

$4y^2 - 96y + 512 = 0$

$y^2 - 24y + 128 = 0$

$(y - 16)(y - 8) = 0$

$y = \dfrac{24 \pm \sqrt{576 - 4(128)}}{2}$

An attempt to solve the quadratic in $y$ by either factorising or by the formula or by *completing the square*. **dM1**

$y = 16$ or $y = 8$.

Both $\underline{y = 16}$ and $\underline{y = 8}$. or $\underline{(-8, 8)}$ and $\underline{(-8, 16)}$. **A1**

**[3]**

(b) $\left\{ \dfrac{\cancel{dy}}{\cancel{dx}} \times \right\}$  $3x^2 - 8y\dfrac{dy}{dx}; = \left( \underline{12y + 12x\dfrac{dy}{dx}} \right)$

Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $12x\frac{dy}{dx}$. Ignore $\frac{dy}{dx} = \ldots$ **M1**

Correct LHS equation; **A1;**

Correct application of product rule **(B1)**

$\left\{ \dfrac{dy}{dx} = \dfrac{3x^2 - 12y}{12x + 8y} \right\}$

*not necessarily required.*

@ $(-8, 8)$,  $\dfrac{dy}{dx} = \dfrac{3(64) - 12(8)}{12(-8) + 8(8)} = \dfrac{96}{-32} = \underline{-3}$,

Substitutes $x = -8$ and *at least one* of their $y$-values to attempt to find any one of $\frac{dy}{dx}$. **dM1**

@ $(-8, 16)$,  $\dfrac{dy}{dx} = \dfrac{3(64) - 12(16)}{12(-8) + 8(16)} = \dfrac{0}{32} = \underline{0}$.

One gradient found. **A1**

Both gradients of $\underline{-3}$ and $\underline{0}$ *correctly* found. **A1 cso**

**[6]**

**9 marks**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **_Aliter_** **5.** (b) **Way 2** | $\left\{\dfrac{\cancel{\mathrm{d}x}}{\cancel{\mathrm{d}y}}\times\right\}$  $3x^2\dfrac{\mathrm{d}x}{\mathrm{d}y}-8y; = \left(\underline{12y\dfrac{\mathrm{d}x}{\mathrm{d}y}+12x}\right)$ | Differentiates implicitly to include either $\pm kx^2\frac{\mathrm{d}x}{\mathrm{d}y}$ or $12y\frac{\mathrm{d}x}{\mathrm{d}y}$. Ignore $\frac{\mathrm{d}x}{\mathrm{d}y}=...$ <br><br> Correct LHS equation <br> <u>Correct application of product rule</u> | M1 <br><br> A1; <br> (B1) |
| | $\left\{\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3x^2-12y}{12x+8y}\right\}$ | _not necessarily required._ | |
| | @ $(-8,\,8)$,  $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{3(64)-12(8)}{12(-8)+8(8)} = \dfrac{96}{-32} = \underline{-3}$, <br><br> @ $(-8,\,16)$,  $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{3(64)-12(16)}{12(-8)+8(16)} = \dfrac{0}{32} = \underline{0}.$ | Substitutes $x=-8$ and _at least one_ of their $y$-values to attempt to find any one of $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$. <br><br> One gradient found. <br> Both gradients of <u>-3</u> and <u>0</u> _correctly_ found. | dM1 <br><br><br> A1 <br> A1 **cso** <br> **[6]** |

| Question Number | Scheme | Marks |
|---|---|---|
| **Aliter**<br>**5.** (b)<br>**Way 3** | $x^3 - 4y^2 = 12xy$ ( eqn $*$ )<br><br>$4y^2 + 12xy - x^3 = 0$<br><br>$y = \dfrac{-12x \pm \sqrt{144x^2 - 4(4)(-x^3)}}{8}$<br><br>$y = \dfrac{-12x \pm \sqrt{144x^2 + 16x^3}}{8}$<br><br>$y = \dfrac{-12x \pm 4\sqrt{9x^2 + x^3}}{8}$<br><br>$y = -\tfrac{3}{2}x \pm \tfrac{1}{2}\left(9x^2 + x^3\right)^{\frac{1}{2}}$<br><br>$\dfrac{dy}{dx} = -\tfrac{3}{2} \pm \tfrac{1}{2}\left(\tfrac{1}{2}\right)\left(9x^2 + x^3\right)^{-\frac{1}{2}};\left(18x + 3x^2\right)$<br><br>$\dfrac{dy}{dx} = -\dfrac{3}{2} \pm \dfrac{18x + 3x^2}{4(9x^2 + x^3)^{\frac{1}{2}}}$<br><br>@ $x = -8$ $\dfrac{dy}{dx} = -\dfrac{3}{2} \pm \dfrac{18(-8) + 3(64)}{4(9(64) + (-512))^{\frac{1}{2}}}$<br><br>$= -\dfrac{3}{2} \pm \dfrac{48}{4\sqrt{(64)}} = -\dfrac{3}{2} \pm \dfrac{48}{32}$<br><br>$\therefore \dfrac{dy}{dx} = -\dfrac{3}{2} \pm \dfrac{3}{2} = \underline{-3},\ \underline{0}.$ | A credible attempt to make $y$ the subject and an attempt to differentiate either $-\tfrac{3}{2}x$ or $\tfrac{1}{2}\left(9x^2 + x^3\right)^{\frac{1}{2}}$.  **M1**<br><br>$\dfrac{dy}{dx} = -\tfrac{3}{2} \pm k\left(9x^2 + x^3\right)^{-\frac{1}{2}}\left(g(x)\right)$  **A1**<br><br>$\dfrac{dy}{dx} = -\tfrac{3}{2} \pm \tfrac{1}{2}\left(\tfrac{1}{2}\right)\left(9x^2 + x^3\right)^{-\frac{1}{2}};\left(18x + 3x^2\right)$  **A1**<br><br>Substitutes $x = -8$ find any one of $\tfrac{dy}{dx}$.  **dM1**<br><br>One gradient correctly found.  **A1**<br>Both gradients of $\underline{-3}$ and $\underline{0}$ correctly found.  **A1**<br>**[6]** |

| Question Number | Scheme | Marks |
|---|---|---|
| **6.** (a) | $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix}$ & $\overrightarrow{OB} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$ | |

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

Finding the difference between $\overrightarrow{OB}$ and $\overrightarrow{OA}$.  M1 $\pm$

Correct answer.  A1

**[2]**

(b)

$l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

$l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$

An expression of the form $\left( \text{vector} \right) \pm \lambda \left( \text{vector} \right)$  M1

$\mathbf{r} = \overrightarrow{OA} \pm \lambda \left( \text{their } \overrightarrow{AB} \right)$ or

$\mathbf{r} = \overrightarrow{OB} \pm \lambda \left( \text{their } \overrightarrow{AB} \right)$ or  A1 $\sqrt{}$ aef

$\mathbf{r} = \overrightarrow{OA} \pm \lambda \left( \text{their } \overrightarrow{BA} \right)$ or

$\mathbf{r} = \overrightarrow{OB} \pm \lambda \left( \text{their } \overrightarrow{BA} \right)$

( $\mathbf{r}$ is needed.)

**[2]**

(c)

$l_2 : \mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \Rightarrow \quad \mathbf{r} = \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$\overrightarrow{AB} = \mathbf{d}_1 = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ , $\mathbf{d}_2 = \mathbf{i} + 0\mathbf{j} + \mathbf{k}$ & $\theta$ is angle

$\cos \theta = \dfrac{\overrightarrow{AB} \bullet \mathbf{d}_2}{\left( \left| \overrightarrow{AB} \right| . \left| \mathbf{d}_2 \right| \right)} = \dfrac{\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}{\left( \sqrt{(1)^2 + (-2)^2 + (2)^2} . \sqrt{(1)^2 + (0)^2 + (1)^2} \right)}$

Considers dot product between $\mathbf{d}_2$ and their $\overrightarrow{AB}$.  M1 $\sqrt{}$

$\cos \theta = \dfrac{1 + 0 + 2}{\sqrt{(1)^2 + (-2)^2 + (2)^2} . \sqrt{(1)^2 + (0)^2 + (1)^2}}$

Correct followed through expression or **equation**.  A1 $\sqrt{}$

$\cos \theta = \dfrac{3}{3.\sqrt{2}} \Rightarrow \underline{\theta = 45° \text{ or } \frac{\pi}{4} \text{ or awrt } 0.79.}$

$\underline{\theta = 45° \text{ or } \frac{\pi}{4} \text{ or awrt } 0.79}$  A1 **cao**

**[3]**

This means that $\cos \theta$ does not necessarily have to be the subject of the equation. It could be of the form $3\sqrt{2} \cos \theta = 3$.

| Question Number | Scheme | Marks |
|---|---|---|
| **6.** (d) | If $l_1$ and $l_2$ intersect then: $\begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ | |

$\mathbf{i}:\quad 2 + \lambda = \mu \qquad (1)$

$\mathbf{j}:\quad 6 - 2\lambda = 0 \qquad (2)$

$\mathbf{k}: -1 + 2\lambda = \mu \qquad (3)$

**Either** seeing equation (2) written down correctly with or without any other equation **or** seeing equations (1) and (3) written down correctly. — M1√

(2) yields $\lambda = 3$

Any two yields $\lambda = 3, \quad \mu = 5$

Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find … — dM1

either one of $\lambda$ or $\mu$ correct. — A1

$l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + 3\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$ *or* $\mathbf{r} = 5\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$

$\begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$ or $5\mathbf{i} + 5\mathbf{k}$ — A1 **cso**

Fully correct solution & no incorrect values of $\lambda$ or $\mu$ seen earlier.

**[4]**

| | | |
|---|---|---|
| **Alite**r **6.** (d) **Way 2** | If $l_1$ and $l_2$ intersect then: $\begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ | |

$\mathbf{i}:\quad 3 + \lambda = \mu \qquad (1)$

$\mathbf{j}:\quad 4 - 2\lambda = 0 \qquad (2)$

$\mathbf{k}:\quad 1 + 2\lambda = \mu \qquad (3)$

**Either** seeing equation (2) written down correctly with or without any other equation **or** seeing equations (1) and (3) written down correctly. — M1√

(2) yields $\lambda = 2$

Any two yields $\lambda = 2, \quad \mu = 5$

Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find … either one of $\lambda$ or $\mu$ correct. — dM1 / A1

$l_1 : \mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + 2\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$ *or* $\mathbf{r} = 5\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$

$\begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$ or $5\mathbf{i} + 5\mathbf{k}$ — A1 **cso**

Fully correct solution & no incorrect values of $\lambda$ or $\mu$ seen earlier.

**[4]**

**11 marks**

**Note:** Be careful! $\lambda$ and $\mu$ are not defined in the question, so a candidate could interchange these or use different scalar parameters.

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| Question Number | Scheme | Marks |
|---|---|---|
| *Aliter* **6.** (d) **Way 3** | If $l_1$ and $l_2$ intersect then: $\begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} = \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ | |

$\mathbf{i}: \quad 2 - \lambda = \mu \qquad (1)$

$\mathbf{j}: \quad 6 + 2\lambda = 0 \qquad (2)$

$\mathbf{k}: -1 - 2\lambda = \mu \qquad (3)$

**Either** seeing equation (2) written down correctly with or without any other equation **or** seeing equations (1) and (3) written down correctly. — M1 $\surd$

(2) yields $\lambda = -3$

Any two yields $\lambda = -3, \ \mu = 5$

Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find … — dM1

either one of $\lambda$ or $\mu$ correct. — A1

$l_1: \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} - 3 \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix} \quad or \quad \mathbf{r} = 5 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$

$\begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$ or $5\mathbf{i} + 5\mathbf{k}$ — A1 **cso**

Fully correct solution & no incorrect values of $\lambda$ or $\mu$ seen earlier.

**[4]**

| Question Number | Scheme | Marks |
|---|---|---|
| *Aliter* **6.** (d) **Way 4** | If $l_1$ and $l_2$ intersect then: $\begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} = \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ | |

$\mathbf{i}: \quad 3 - \lambda = \mu \qquad (1)$

$\mathbf{j}: \quad 4 + 2\lambda = 0 \qquad (2)$

$\mathbf{k}: \quad 1 - 2\lambda = \mu \qquad (3)$

**Either** seeing equation (2) written down correctly with or without any other equation **or** seeing equations (1) and (3) written down correctly. — M1 $\surd$

(2) yields $\lambda = -2$

Any two yields $\lambda = -2, \ \mu = 5$

Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find … either one of $\lambda$ or $\mu$ correct. — dM1, A1

$l_1: \mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} - 2 \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix} \quad or \quad \mathbf{r} = 5 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$

$\begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$ or $5\mathbf{i} + 5\mathbf{k}$ — A1 **cso**

Fully correct solution & no incorrect values of $\lambda$ or $\mu$ seen earlier.

**[4]**

**11 marks**

| Question Number | Scheme | Marks |
|---|---|---|
| **7.** (a) | $\left[ x = \ln(t+2), \ \ y = \dfrac{1}{t+1} \right], \qquad \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{1}{t+2}$      Must state $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{1}{t+2}$ | B1 |

$$\text{Area}(R) = \int_{\ln 2}^{\ln 4} \frac{1}{t+1}\,\mathrm{d}x \,; = \int_0^2 \left(\frac{1}{t+1}\right)\!\left(\frac{1}{t+2}\right)\mathrm{d}t$$

Area $= \displaystyle\int \frac{1}{t+1}\,\mathrm{d}x$ .  **M1;**
Ignore limits.

$\displaystyle\int \left(\frac{1}{t+1}\right)\times\left(\frac{1}{t+2}\right)\mathrm{d}t$ . Ignore limits.    **A1 AG**

Changing limits, when:
$x = \ln 2 \ \Rightarrow \ln 2 = \ln(t+2) \ \Rightarrow \ 2 = t+2 \ \Rightarrow t = 0$
$x = \ln 4 \ \Rightarrow \ln 4 = \ln(t+2) \ \Rightarrow \ 4 = t+2 \ \Rightarrow t = 2$

changes limits $x \to t$
so that $\ln 2 \to 0$ and $\ln 4 \to 2$    **B1**

Hence, $\text{Area}(R) = \displaystyle\int_0^2 \frac{1}{(t+1)(t+2)}\,\mathrm{d}t$

**[4]**

(b) $\left(\dfrac{1}{(t+1)(t+2)}\right) = \dfrac{A}{(t+1)} + \dfrac{B}{(t+2)}$     $\dfrac{A}{(t+1)} + \dfrac{B}{(t+2)}$ with $A$ and $B$ found   **M1**

$1 = A(t+2) + B(t+1)$

Let $t = -1, \quad 1 = A(1) \quad \Rightarrow \ \underline{A = 1}$

Let $t = -2, \quad 1 = B(-1) \ \Rightarrow \ \underline{B = -1}$

Finds both $A$ and $B$ correctly.
Can be implied.
(See note below)    **A1**

$\displaystyle\int_0^2 \frac{1}{(t+1)(t+2)}\,\mathrm{d}t = \int_0^2 \frac{1}{(t+1)} - \frac{1}{(t+2)}\,\mathrm{d}t$

$= \Big[ \ln(t+1) - \ln(t+2) \Big]_0^2$

Either $\pm a\ln(t+1)$ or $\pm b\ln(t+2)$    **dM1**
Both ln terms correctly ft.    **A1 $\surd$**

$= \big( \ln 3 - \ln 4 \big) - \big( \ln 1 - \ln 2 \big)$

Substitutes **both** limits of 2 and 0
and subtracts the correct way round.    **ddM1**

$= \ln 3 - \ln 4 + \ln 2 = \ln 3 - \ln 2 = \ln\!\left(\tfrac{3}{2}\right)$

$\underline{\ln 3 - \ln 4 + \ln 2}$ or $\underline{\ln\!\left(\tfrac{3}{4}\right) - \ln\!\left(\tfrac{1}{2}\right)}$
or $\underline{\ln 3 - \ln 2}$ or $\underline{\ln\!\left(\tfrac{3}{2}\right)}$
(must deal with $\ln 1$)    **A1 aef isw**

**[6]**

Takes out brackets.

Writing down $\dfrac{1}{(t+1)(t+2)} = \dfrac{1}{(t+1)} + \dfrac{1}{(t+2)}$ means first M1A0 in (b).

Writing down $\dfrac{1}{(t+1)(t+2)} = \dfrac{1}{(t+1)} - \dfrac{1}{(t+2)}$ means first M1A1 in (b).

| Question Number | Scheme | Marks |
|---|---|---|
| **7.** (c) | $x = \ln(t+2),\quad y = \dfrac{1}{t+1}$ <br><br> $e^x = t+2 \;\Rightarrow\; t = e^x - 2$ <br><br><br><br> $y = \dfrac{1}{e^x - 2 + 1} \;\Rightarrow\; y = \dfrac{1}{e^x - 1}$ | Attempt to make $t = ...$ the subject    M1 <br> giving $t = e^x - 2$    A1 <br><br> Eliminates $t$ by substituting in $y$    dM1 <br> giving $y = \dfrac{1}{e^x - 1}$    A1 <br> **[4]** |
| *Aliter* <br> **7.** (c) <br> **Way 2** | $t + 1 = \dfrac{1}{y} \;\Rightarrow\; t = \dfrac{1}{y} - 1 \;$ or $\; t = \dfrac{1-y}{y}$ <br><br> $y(t+1) = 1 \;\Rightarrow\; yt + y = 1 \;\Rightarrow\; yt = 1 - y \;\Rightarrow\; t = \dfrac{1-y}{y}$ <br><br><br> $x = \ln\left(\dfrac{1}{y} - 1 + 2\right) \quad$ or $\quad x = \ln\left(\dfrac{1-y}{y} + 2\right)$ <br><br> $x = \ln\left(\dfrac{1}{y} + 1\right)$ <br><br> $e^x = \dfrac{1}{y} + 1$ <br><br> $e^x - 1 = \dfrac{1}{y}$ <br><br> $y = \dfrac{1}{e^x - 1}$ | Attempt to make $t = ...$ the subject    M1 <br><br> Giving either $t = \dfrac{1}{y} - 1 \;$ or $\; t = \dfrac{1-y}{y}$    A1 <br><br> Eliminates $t$ by substituting in $x$    dM1 <br><br><br><br><br><br><br><br> giving $y = \dfrac{1}{e^x - 1}$    A1 <br> **[4]** |
| (d) | Domain : <u>$x > 0$</u> | <u>$x > 0$</u> or just $> 0$    B1 <br> **[1]** |
| | | **15 marks** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| ***Aliter*** **7.** (c) **Way 3** | $e^x = t + 2 \implies t + 1 = e^x - 1$ | Attempt to make $t + 1 = ...$ the subject | M1 |
| | | giving $t + 1 = e^x - 1$ | A1 |
| | $y = \dfrac{1}{t+1} \implies y = \dfrac{1}{e^x - 1}$ | Eliminates $t$ by substituting in $y$ | dM1 |
| | | giving $y = \dfrac{1}{e^x - 1}$ | A1 |
| | | | **[4]** |
| ***Aliter*** **7.** (c) **Way 4** | $t + 1 = \dfrac{1}{y} \implies t + 2 = \dfrac{1}{y} + 1$ or $t + 2 = \dfrac{1+y}{y}$ | Attempt to make $t + 2 = ...$ the subject | M1 |
| | | Either $t + 2 = \dfrac{1}{y} + 1$ or $t + 2 = \dfrac{1+y}{y}$ | A1 |
| | $x = \ln\left(\dfrac{1}{y} + 1\right)$ or $x = \ln\left(\dfrac{1+y}{y}\right)$ | Eliminates $t$ by substituting in $x$ | dM1 |
| | $x = \ln\left(\dfrac{1}{y} + 1\right)$ | | |
| | $e^x = \dfrac{1}{y} + 1 \implies e^x - 1 = \dfrac{1}{y}$ | | |
| | $y = \dfrac{1}{e^x - 1}$ | giving $y = \dfrac{1}{e^x - 1}$ | A1 |
| | | | **[4]** |

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**8.** (a)    $\dfrac{dV}{dt}=1600-c\sqrt{h}$   or   $\dfrac{dV}{dt}=1600-k\sqrt{h}$ ,      Either of these statements    M1

$\left(V=4000h \Rightarrow\right)\ \dfrac{dV}{dh}=4000$      $\dfrac{dV}{dh}=4000$ or $\dfrac{dh}{dV}=\dfrac{1}{4000}$    M1

$\dfrac{dh}{dt}=\dfrac{dh}{dV}\times\dfrac{dV}{dt}=\dfrac{\frac{dV}{dt}}{\frac{dV}{dh}}$

Either, $\dfrac{dh}{dt}=\dfrac{1600-c\sqrt{h}}{4000}=\dfrac{1600}{4000}-\dfrac{c\sqrt{h}}{4000}=0.4-k\sqrt{h}$

or    $\dfrac{dh}{dt}=\dfrac{1600-k\sqrt{h}}{4000}=\dfrac{1600}{4000}-\dfrac{k\sqrt{h}}{4000}=0.4-k\sqrt{h}$

Convincing proof of $\dfrac{dh}{dt}$    A1   **AG**

     **[3]**

(b)   When $h=25$ water ***leaks out such that*** $\dfrac{dV}{dt}=400$

$400=c\sqrt{h} \Rightarrow 400=c\sqrt{25} \Rightarrow 400=c(5) \Rightarrow c=80$

From above; $k=\dfrac{c}{4000}=\dfrac{80}{4000}=0.02$ as required      Proof that $k=0.02$    B1   **AG**

     **[1]**

***Aliter***
(b)
**Way 2**    $400=4000k\sqrt{h}$

$\Rightarrow 400=4000k\sqrt{25}$

Using 400, 4000 and $h=25$

$\Rightarrow 400=k(20000) \Rightarrow k=\frac{400}{20000}=0.02$      or $\sqrt{h}=5$. Proof that $k=0.02$    B1   **AG**

     **[1]**

***Separates the variables*** with

(c)   $\dfrac{dh}{dt}=0.4-k\sqrt{h} \Rightarrow \displaystyle\int\dfrac{dh}{0.4-k\sqrt{h}}=\int dt$      $\displaystyle\int\dfrac{dh}{0.4-k\sqrt{h}}$ and $\displaystyle\int dt$ on either side    M1 oe

with integral signs not necessary.

$\therefore$ time required $=\displaystyle\int_{0}^{100}\dfrac{1}{0.4-0.02\sqrt{h}}\,dh \quad \begin{array}{c}\div 0.02\\ \div 0.02\end{array}$

time required $=\displaystyle\int_{0}^{100}\dfrac{50}{20-\sqrt{h}}\,dh$      Correct proof    A1   **AG**

     **[2]**

| Question Number | Scheme | Marks |
|---|---|---|
| **8.** (d) | $\displaystyle\int_0^{100} \frac{50}{20-\sqrt{h}}\,\mathrm{d}h$ with substitution $h=(20-x)^2$ | |

$$\frac{\mathrm{d}h}{\mathrm{d}x}=2(20-x)(-1) \quad\text{or}\quad \frac{\mathrm{d}h}{\mathrm{d}x}=-2(20-x)$$

Correct $\dfrac{\mathrm{d}h}{\mathrm{d}x}$    B1 **aef**

$$h=(20-x)^2 \Rightarrow \sqrt{h}=20-x \Rightarrow x=20-\sqrt{h}$$

$$\int \frac{50}{20-\sqrt{h}}\,\mathrm{d}h = \int \frac{50}{x}.-2(20-x)\,\mathrm{d}x$$

$\pm\lambda\displaystyle\int \frac{20-x}{x}\,\mathrm{d}x$ or

$\pm\lambda\displaystyle\int \frac{20-x}{20-(20-x)}\,\mathrm{d}x$    M1

where $\lambda$ is a constant

$$=100\int \frac{x-20}{x}\,\mathrm{d}x$$

$$=100\int \left(1-\frac{20}{x}\right)\,\mathrm{d}x$$

$$=100\big(x-20\ln x\big)\ (+c)$$

$\pm\alpha x \pm \beta\ln x\,;\ \alpha,\beta\neq0$    M1

$100x-2000\ln x$    A1

change limits: when $h=0$ then $x=20$
and when $h=100$ then $x=10$

$$\int_0^{100} \frac{50}{20-\sqrt{h}}\,\mathrm{d}h = \Big[100x-2000\ln x\Big]_{20}^{10}$$

or $\displaystyle\int_0^{100} \frac{50}{20-\sqrt{h}}\,\mathrm{d}h = \Big[100\big(20-\sqrt{h}\big)-2000\ln\big(20-\sqrt{h}\big)\Big]_0^{100}$

Correct use of limits, ie. putting them in the correct way round
Either $x=10$ and $x=20$
or $h=100$ and $h=0$    ddM1

$$=\big(1000-2000\ln 10\big)-\big(2000-2000\ln 20\big)$$

$$=2000\ln 20 - 2000\ln 10 - 1000$$

Combining logs to give...

$2000\ln 2 - 1000$

$$=2000\ln 2 - 1000$$

or $-2000\ln\!\left(\tfrac12\right)-1000$    A1 **aef**

**[6]**

(e)    Time required $=2000\ln 2 - 1000 = 386.2943611...$ sec

$=386$ seconds (nearest second)

$=6$ minutes and $26$ seconds (nearest second)

<u>6 minutes, 26 seconds</u>    B1

**[1]**

**13 marks**